Semester IV

GE-4: Numerical Methods   **OR**   **GE-4: Elements of Analysis**
       (with practicals)

GE-4: Numerical Methods

Total Marks: 150 (Theory: 75, Internal Assessment: 25 and Practical: 50)
Workload: 4 Lectures (per week), 4 Practicals (per week per student)
Duration: 14 Weeks (56 Hrs. Theory + 56 Hrs. Practical) **Examination:** 3 Hrs.

Course Objectives: The goal of this paper is to acquaint students for the study of certain algorithms that uses numerical approximation for the problems of mathematical analysis. Also, the use of Computer Algebra Systems (CAS) by which the intractable problems can be solved both numerically and analytically.

Course Learning Outcomes: After completion of this course, students will be able to:
  i) Find the consequences of finite precision and the inherent limits of numerical methods.
  ii) Appropriate numerical methods to solve algebraic and transcendental equations.
  iii) How to solve first order initial value problems of ODE's numerically using Euler methods.

Course Contents:

Unit 1: Errors and Roots of Transcendental and Polynomial Equations   **(Lectures: 16)**
Floating point representation and computer arithmetic, Significant digits; Errors: Roundoff error, Local truncation error, Global truncation error; Order of a method, Convergence and terminal conditions; Bisection method, Secant method, Regula-Falsi method, Newton-Raphson method.

Unit 2: Algebraic Linear Systems and Interpolation   **(Lectures: 20)**
Gaussian elimination method (with row pivoting), Gauss-Jordan method; Iterative methods: Jacobi method, Gauss-Seidel method; Interpolation: Lagrange form, Newton form, Finite difference operators, Gregory-Newton forward and backward difference interpolations, Piecewise polynomial interpolation (Linear and Quadratic).

Unit 3: Numerical Differentiation, Integration and ODE   **(Lectures: 20)**
Numerical differentiation: First and second order derivatives; Numerical integration: Trapezoid rule, Simpson's rule; Extrapolation methods: Richardson extrapolation, Romberg integration; Ordinary differential equation: Euler's method, Modified Euler's methods (Heun and Mid-point).

References:

1. Chapra, Steven C. (2018). *Applied Numerical Methods with* MATLAB *for Engineers and Scientists* (4th ed.). McGraw-Hill Education.
2. Fausett, Laurene V. (2009). *Applied Numerical Analysis Using* MATLAB. Pearson. India.

3. Jain, M. K., Iyengar, S. R. K., & Jain R. K. (2012). *Numerical Methods for Scientific and Engineering Computation* (6th ed.). New Age International Publishers. Delhi.

Additional Reading:

1. Bradie, Brian (2006). *A Friendly Introduction to Numerical Analysis*. Pearson Education India. Dorling Kindersley (India) Pvt. Ltd. Third Impression, 2011.

Practical /Lab work to be performed in the Computer Lab:

Use of Computer Algebra System (CAS), for example MATLAB/Mathematica/Maple/Maxima/ Scilab etc., for developing the following Numerical Programs:

1) Bisection Method
2) Secant Method and Regula-Falsi Method
3) Newton-Raphson Method
4) Gaussian elimination method and Gauss-Jordan method
5) Jacobi Method and Gauss-Seidel Method
6) Lagrange Interpolation and Newton Interpolation
7) Trapezoid and Simpson's rule.
8) Romberg integration
9) Euler methods for solving first order initial value problems of ODE's.

Teaching Plan (Theory of GE-4: Numerical Methods):

Weeks 1 and 2: Floating point representation and computer arithmetic, Significant digits; Errors: Roundoff error, Local truncation error, Global truncation error; Order of a method, Convergence and terminal conditions.
[2] Chapter 1 (Sections 1.2.3, 1.3.1, and 1.3.2)
[3] Chapter 1 (Sections 1.2, and 1.3)

Week 3 and 4: Bisection method, Secant method, Regula-Falsi method, Newton-Raphson method.
[2] Chapter 2 (Sections 2.1 to 2.3)
[3] Chapter 2 (Sections 2.2 and 2.3)

Week 5: Gaussian elimination method (with row pivoting), Gauss-Jordan method;
Iterative methods: Jacobi method, Gauss-Seidel method.
[2] Chapter 3 (Sections 3.1, and 3.2), Chapter 6 (Sections 6.1, and 6.2)
[3] Chapter 3 (Sections 3.2, and 3.4)

Week 6: Interpolation: Lagrange form, and Newton form.
[2] Chapter 8 (Section 8.1)
[3] Chapter 4 (Section 4.2)

Weeks 7 and 8: Finite difference operators, Gregory-Newton forward and backward difference interpolations.
[3] Chapter 4 (Sections 4.3, and 4.4)

Week 9: Piecewise polynomial interpolation: Linear, and Quadratic.
[2] Chapter 8 [Section 8.3 (8.3.1, and 8.3.2)]
[1] Chapter 18 (Sections 18.1 to 18.3)

Weeks 10 and 11: Numerical differentiation: First and second order derivatives;
Numerical integration: Trapezoid rule, Simpson's rule.
[2] Chapter 11 [Sections 11.1 (11.1.1, and 11.1.2), and 11.2 (11.2.1, and 11.2.2)]

Weeks 12 and 13: Extrapolation methods: Richardson extrapolation, Romberg integration;
Ordinary differential equations: Euler's method.
[2] Chapter 11 [Section 11.1 (11.1.4), and 11.2 (11.2.4)]
[1] Chapter 22 (Sections 22.1, and 22.2)

Weeks 14: Modified Euler's methods: Heun's method, The Midpoint method.
[1] Chapter 22 (Section 22.3)