BMS- III Sem.

[This question paper contains 6 printed pages.]

**Sr. No. of Question Paper : 1635** **F-3** **Your Roll No.....**

Unique Paper Code : 1091301

Name of the Course : **B.M.S.**

Name of the Paper : Operations Research

Semester : III

Duration : 3 Hours Maximum Marks : 75

**Instructions for Candidates**

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. This question paper contains **five** printed sheets.
3. Answer **any five** questions.
4. Attempt **all** parts of a question together.
5. Show your workings clearly on the answer sheet itself.
6. Use of Simple Calculator is allowed.

1. A diet for a sick person must contain at least 4,000 units of vitamins, 50 units of minerals and 1400 units of calories. Two foods A and B are available at a cost of Rs. 4 and Rs. 3 per unit respectively. One unit of A contains 200 units of vitamins, 1 unit of mineral and 40 units of calories, while one unit of food B contains 100 units of vitamins, 2 units of minerals and 40 units of calories.

   (i) Formulate the above problem as LPP. (3)

   (ii) Standardise the above problem by introducing slack, surplus or artificial variables as required. Give justification for the same. (3)

   (iii) Solve the above problem using Simplex method and find the combination of food A and B that should be given to the sick person in order to provide required vitamins, minerals and calories at least cost. (8)

   (iv) Is the solution unique ? Give reason for your answer. (1)

*P.T.O.*

2. (a) Discuss the concept of Duality in LPP. Write the dual for the following LPP.

Maximize $Z = 3x_1 + 5x_2 + 7x_3$

Subject to :

$$x_1 + x_2 + 3x_3 \leq 10$$

$$4x_1 - x_2 + 2x_3 \geq 15 \qquad (3,5)$$

$x_1, x_2 \geq 0$ and $x_3$ is unrestricted in sign.

(b) A company manufactures and sells 3 models of pressure cookers $X_1$, $X_2$ and $X_3$ that use two resources, aluminium and machine time. While market demand for three models poses no constraints, supplies of aluminium is limited to 750 kg per week and availability of machine time is limited to 600 hours per week. The resource usage of the 3 models and their profitability are given below :

| | $X_1$ | $X_2$ | $X_3$ |
|---|---|---|---|
| Aluminium/unit | 6 | 3 | 5 |
| Machine-time/unit | 3 | 4 | 5 |
| Contribution/unit | 60 | 20 | 80 |

The final optimum solution to the above LPP is given as follows :

| Basic Variable | $C_j$ | $X_1$ | $X_2$ | $X_3$ | $S_1$ | $S_2$ | Solution |
|---|---|---|---|---|---|---|---|
| $X_1$ | 60 | 1 | –1/3 | 0 | 1/3 | –1/3 | 50 |
| $X_3$ | 80 | 0 | 1 | 1 | –1/5 | 2/5 | 90 |
| $C_j$ | | 60 | 20 | 80 | 0 | 0 | |
| $Z_j$ | | 60 | 60 | 80 | 4 | 12 | |
| $C_j - Z_j$ | | 0 | –40 | 0 | –4 | –12 | |

Using the information given in the above table, determine whether and how the current solution would be sensitive to the following changes. Treat each of the conditions given below independently.

(i) An additional 150 kg of aluminium would become available.

(ii) The machine hours available would reduce from the current level of 600 hours to 450 hours.

(iii) A new model has been developed requiring 3 kg of aluminium and 3 hours of machine time per unit, with an estimated unit contribution of Rs. 40. Would it be profitable to manufacture this model ?

(iv) In context of the above changes, comment on the significance of sensitivity analysis. (7)

3. (a) Discuss Integer Programming Problem. Differentiate between a pure and a mixed Integer Programming Problem. (3)

(b) The following transportation problem gives a degenerate basic solution (i.e., at least one of the basic variables is zero). Assume that the solution is optimal and the associated values of the multipliers are $u_1 = 1$, $u_2 = -1$, $v_1 = 2$, $v_2 = 2$ and $v_3 = 5$; and cell entries represent $x_{ij}$'s i.e. number of items to be transported from $i^{th}$ origin to $j^{th}$ destination.

| Origin | Destinations | | | Availability |
|---|---|---|---|---|
| | $D_1$ | $D_2$ | $D_3$ | |
| 1 | 10 | | | 10 |
| 2 | 0 | 20 | 20 | 40 |
| Requirement | 10 | 20 | 20 | |

(i) Find the associated optimal cost for the above solution.

(ii) Determine the smallest value of $c_{ij}$ associated with each non-basic variable that will maintain the optimality of the above solution. (2,2)

(c) In the following transportation problem, the total demand exceeds the total supply. Suppose that the penalty costs per unit of unsatisfied demands are Rs. 5, 3 and 2 respectively. Find the optimum solution and the associated total cost.

| Origin | Destinations | | | Availability |
|---|---|---|---|---|
| | P | Q | R | |
| A | 5 | 1 | 7 | 10 |
| B | 6 | 4 | 6 | 80 |
| C | 3 | 2 | 5 | 15 |
| Demand | 75 | 20 | 50 | |

(8)

4. (a) Define total, free and independent float of an activity. (3)

(b) The table below gives data on normal and shortest time for completing each activity of a building contract and the cost per day for reducing the time for each activity. The contract includes a penalty cost of Rs. 100 per day over 17 days. The overhead cost per day is Rs. 160. The cost of completing the eight activities in normal time is Rs. 6500.

(i) Draw the network and identify the critical path.

(ii) What is the normal project duration and associated costs ?

(iii) Crash the relevant activities systematically and determine the optimum project completion time and cost.

| Activity | Normal Time (Days) | Shortest Time | Cost of reduction per day |
|---|---|---|---|
| 1-2 | 6 | 4 | 80 |
| 1-3 | 8 | 4 | 90 |
| 1-4 | 5 | 3 | 30 |
| 2-4 | 3 | 3 | – |
| 2-5 | 5 | 3 | 40 |
| 3-6 | 12 | 8 | 200 |
| 4-6 | 8 | 5 | 50 |
| 5-6 | 6 | 6 | – |

(4,2,6)

5. (a) The personnel manager of a medium-sized company has decided to recruit two employees D and E in a particular section of the organization. The section has five fairly defined tasks 1, 2, 3, 4 and 5; and three employees A, B and C are already employed in the section. Considering the specialized nature of task 3 and the special qualification of the recruit D for task 3, the manager has decided to assign task 3 to employee D and then assign the remaining tasks to remaining employees so as to maximize the total effectiveness. The index of effectiveness of each employee for different tasks is as under :

| Employees \ Tasks | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| A | 25 | 55 | 60 | 45 | 30 |
| B | 45 | 65 | 55 | 35 | 40 |
| C | 10 | 35 | 45 | 55 | 65 |
| D | 40 | 30 | 70 | 40 | 60 |
| E | 55 | 45 | 40 | 55 | 10 |

Assign the tasks for maximizing total effectiveness. (8)

(b) Use the graphical method for solving the following game and find the value of the game. (The payoff matrix is for player A)

| | Player B | | | |
|---|---|---|---|---|
| Player A | $B_1$ | $B_2$ | $B_3$ | $B_4$ |
| $A_1$ | 2 | 2 | 3 | –2 |
| $A_2$ | 4 | 3 | 2 | 6 |

(7)

6. (a) A grocery with a bakery department is faced with the problem of how many cakes to buy in order to meet the day's demand. The grocer prefers not to sell day-old goods in competition with fresh products. Leftover cakes are therefore a complete toss. On the other hand, if a customer desires a cake and all of them have been sold, the disappointed customer will buy elsewhere and the sales will be lost. The grocer has therefore collected information on the past sales on a selected 100 day period as shown in the table below :

| Sales per day | No. of days | Probability |
|---|---|---|
| 25 | 10 | 0.10 |
| 26 | 30 | 0.30 |
| 27 | 50 | 0.50 |
| 28 | 10 | 0.10 |

A cake costs Rs. 80 and sells for Rs. 100. Construct the payoff table and the opportunity loss table. What is the optimum number of cakes that should be bought each day ? (8)

(b) A certain piece of equipment is inspected at the end of each day and classified as just overhauled, good, fair, or inoperative. The four classifications are denoted as 1, 2, 3 and 4 respectively. If an item is inoperative it is overhauled, a procedure that takes one day. Assume that the working condition of the equipment follows a Markov process with the following state-transition matrix :

| | | **Tomorrow** | | | |
|---|---|---|---|---|---|
| | | **1** | **2** | **3** | **4** |
| **Today** | **1** | 0 | 3/4 | 1/4 | 0 |
| | **2** | 0 | 1/2 | 1/2 | 0 |
| | **3** | 0 | 0 | 1/2 | 1/2 |
| | **4** | 1 | 0 | 0 | 0 |

It costs Rs. 125 to overhaul a machine on an average, and Rs. 75 is lost in production if a machine is found inoperative. Use the steady state probabilities to compute the expected per day cost of maintenance. (7)